Statistical Estimation via Convex Optimization

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SCHEDULE	Thursday	18th October 2018	De 14h à 16h30	Salle 2043
	Wednesday	7th November 2018	**De 14h à 16h30 Changement**	Salle 2043
	Friday	19th October 2018 9th November 2018	De 14h à 16h30	Salle 2043

When speaking about links between Statistics and Optimization, what comes to mind first is the indispensable role played by optimization algorithms in the "numerical toolbox" of Statistics. The goal of this course is to present another type of links between Optimization and Statistics. We are speaking of situations where Optimization theory (theory, not algorithms!) is of methodological value in Statistics, acting as the source of statistical inferences. We focus on utilizing Convex Programming theory, mainly due to its power, but also due to the desire to end up with inference routines reducing to solving convex optimization problems and thus implementable in a *computationally efficient fashion*.
The topics we consider are:

- As a starter, we consider estimation of a linear functional of unknown "signal" (a signal in the "usual sense," a distribution, or an intensity of a Poisson process, etc). We also discuss the problem of estimating quadratic functional by "lifting" linear functional estimates. As an application, we consider a signal recovery procedure – "polyhedral estimate" – which relies upon efficient estimation of linear functionals.
- Next, we turn to general problem of linear estimation of signals from noisy observations of their linear images. Here application of Convex Optimization allows to propose provably optimal (or nearly so) estimation procedures.

The exposition does *not* require prior knowledge of Statistics and Optimization; as far as these disciplines are concerned, all necessary for us facts and concepts are introduced before being used. The actual prerequisites are elementary Calculus, Probability, Linear Algebra and (last but by far not least) general mathematical culture.

References:

- J. Kuks and W. Olman. Minimax linear estimation of regression coefficients (I). *Iswestija Akademii Nauk Estonskoj SSR*, 20:480–482, 1971.
- M. Pinsker. Optimal filtration of square-integrable signals in Gaussian noise. *Prob. Info. Trans.*, 16(2): 120–133, 1980
- I.A. Ibragimov, R.Z. Khas'minskij. On the nonparametric estimation of a value of a linear functional in Gaussian white noise. *Teor. Veroyatnost. i Primenen.* 29:19–32, 1984. (Russian. English summary.)
- D. Donoho, R. Liu, and B. MacGibbon. Minimax risk over hyperrectangles, and implications. *Ann. Statist.* 18:1416–1437, 1990.
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- A. Ben-Tal, A. Nemirovski, *Lectures on Modern Convex Optimization*, Book version: MPS-SIAM Series on Optimization, SIAM, Philadelphia, 2001
- J. A. Tropp. An introduction to matrix concentration inequalities. *Foundations and Trends in Machine Learning*, 8(1-2):1–230, 2015
- A. Juditsky, A. Nemirovski. *Statistical Inferences via Convex Optimization* (Lecture Notes)

à l'ENSAE, - 5 Av. Henry Le Chatelier - Palaiseau (REB B Massy Palaiseau & bus 9106 Cou B)
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