

CREST - GENES

Cours doctoraux 2019 – 2020

Estimation of Functionals of High-Dimensional Parameters: Bias Reduction and Concentration

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SCHEDULE	Monday	04th May 2020 11th May 2020 18th May 2020	De 14h à 16h De 14h à 17h De 14h à 16h	Salle 2001
	Thursday	07th May 2020 14th May 2020	De 14h à 16h	Salle 2001

Summary :

The main focus of this course will be on a circle of problems related to estimation of real valued functionals of parameters of high-dimensional statistical models. In such problems, it is of interest to estimate one-dimensional features of a high-dimensional parameter that are often represented by nonlinear functionals of certain degree of smoothness defined on the parameter space. The functionals of interest could be estimated with faster convergence rates than the whole parameter (sometimes, even with parametric rates). Potential examples include, for instance, such problems as estimation of linear functionals of principal components (that are nonlinear functionals of unknown covariance) in high-dimensional PCA. The goal is to discuss several mathematical methods that provide a way to develop estimators of smooth functionals of high-dimensional parameters with parametric \sqrt{n} convergence rates and to prove their asymptotic efficiency.

The following topics will be covered (at least, to some extent):

- preliminaries in high-dimensional probability and analysis (concentration inequalities, comparison inequalities, smoothness of operator functions, etc);
- non-asymptotic bounds and concentration inequalities for sample covariance in high-dimensional and dimension-free frameworks;
- some approaches to concentration inequalities for smooth functionals of statistical estimators;
- a recent approach to bias reduction in functional estimation based on an approximate solution of integral equations with respect to Markov kernels on the parameter space;
- a coupling method (based on representations of Markov chains as superpositions of independent "random homotopies") that allows one to obtain bounds on the bias of the resulting estimators;
- minimax lower bounds in functional estimation based on van Trees inequality and on Nemirovski's method.

Literature:

R. Adamczak and P. Wolffe, Concentration inequalities for non-Lipschitz functions with bounded derivatives of higher orders, *Probability Theory and Related Fields*, 2015, 162 (3-4), 531-586.

V. Koltchinskii, M. Loffler and R. Nickl, Efficient Estimation of Linear Functionals of Principal Components, *Annals of Statistics*, 2019, to appear arXiv:1708.07642.

V. Koltchinskii, Asymptotically Efficient Estimation of Smooth Functionals of Covariance Operators, *J. European Mathematical Society*, 2019, to appear arXiv:1710.09072.

V. Koltchinskii and M. Zhilova, Estimation of Smooth Functionals in Normal Models: Bias Reduction and Asymptotic Efficiency, 2019, in preparation.

A. Nemirovski, Topics in Non-Parametric Statistics, in: M. Emery, A. Nemirovski, D. Voiculescu, *Lectures on Probability Theory and Statistics*, Ecole d'ete de Probabilités de Saint-Flour, XXVIII - 1998, Editor: P. Bernard, *Lecture Notes in Mathematics*, 1738, Springer, 2000.

R. Vershynin, *High-Dimensional Probability. An Introduction with Applications in Data Science*, Cambridge University Press, 2018.

à l'ENSAE, - 5 Av. Henry Le Chatelier - Palaiseau (REB B Massy Palaiseau & bus 9106 C ou B)

Ces cours sont proposés aux étudiants de 3^{ème} année de l'ENSAE, de l'ENSAI, ouverts aux étudiants de M2 ou inscrits en thèse. **Une inscription préalable est demandée impérativement** pour tous les étudiants de l'ENSAE, de l'ENSAI, ou extérieurs, à Lyza RACON : lyza.racon@ensae.fr ou par téléphone au 0170266926 afin de pouvoir être admis dans les locaux de l'ENSAE et pouvoir être joints en cas de nécessité par les organisateurs du cours.